THE CALCULATION OF SOUND LEVELS CAUSED BY REFLECTIONS AT CYLINDRICAL SURFACES

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Introduction

Prediction of sound levels and generally the calculation of sound propagation can be performed using the methodology of ISO 9613-2[1]. This semi-empirical method is often criticised and qualified as being out-of-date, but it is nevertheless clear and transparent and compared with more modern concepts like Nord2000, Harmonoise or SonRoad by far more reproducible and precise. But similar with other methods it shows some weaknesses that should be eliminated step by step.
One of these weaknesses is the calculation of reflections at cylindrical or generally at curved surfaces as it is necessary with tanks, vessels, ovens or silos.The calculation method for such reflections described in ISO 9613-2 originally taken from the “Nordic Prediction Method” is related to very specific source – receiver positions and cannot be used generally e. g. in noise mapping projects.
In the following a solution is derived and presented that is based on a geometrical ray concept and that can seamlessly be integrated in the methodology of ISO 9613-2 or in similar concepts.

Reflection of plane sound wave at a vertical cylinder

A sound ray propagating in negative y-direction in a distance a parallel to the y- axis may strike the cylindrical surface as it is shown in Figure 1
From the equation of the circle

$$x^2 + y^2 = R^2$$

or

$$y = \sqrt{R^2 - x^2} \qquad (1)$$

the gradient of the tangent at the reflection point is

$$y' = m_T = \frac{-1}{\sqrt{\left(\frac{1}{k}\right)^2 - 1}} \qquad (2)$$

with
$$k = x_{ref} / R \tag{3}$$

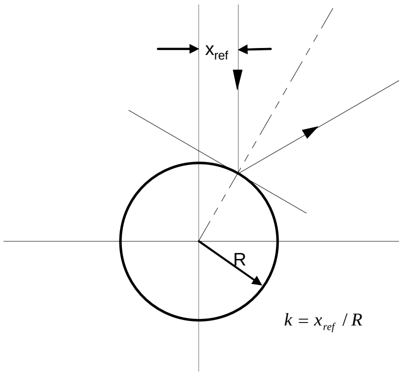

Figure 1 Geometry of a ray reflected at the cylindrical shell

The gradient of the tangent normal is
$$m_N = -\frac{1}{y'} = \sqrt{\left(\frac{1}{k}\right)^2 - 1} \tag{4}$$
the angle with the x-axis
$$\alpha_N = \arctan\sqrt{\left(\frac{1}{k}\right)^2 - 1} \tag{5}$$
and the angle between the inbound ray and the tangents normal
$$\alpha^* = \frac{\pi}{2} - \arctan\left(\sqrt{\left(\frac{1}{k}\right)^2 - 1}\right) \tag{6}$$
The gradient of the reflected ray or ist angle with the x-axis is therefore
$$\alpha_{ref} = \alpha_N - \left(\frac{\pi}{2} - \arctan\left(\sqrt{\left(\frac{1}{k}\right)^2 - 1}\right)\right) = 2\arctan\left(\sqrt{\left(\frac{1}{k}\right)^2 - 1}\right) - \frac{\pi}{2} \tag{7}$$
With a width of the inbound ray dx the aperture angle of the reflected ray is $d\alpha$. This angle follows from the dependency between differential shift of the inbound ray in x-direction and the gradient of the reflected ray α_{ref}.
$$d\alpha = \left|\frac{\partial\alpha}{\partial x}\right| \cdot dx \tag{8}$$
Introducing (5) and differentiation results in
$$d\alpha = \frac{2 \cdot dx}{R \cdot \sqrt{1 - k^2}} \tag{9}$$
The width of the reflected ray directly at the cylindrical surface is also dx. This reflected ray with an aperture angle $d\alpha$ can be continued virtually backwards to be radiated from a mirror image source in a distance a behind the reflecting surface element. This distance a can be derived from the condition

$$a \cdot d\alpha = dx$$

and by introducing (9) this results in

$$a = \frac{R \cdot \sqrt{1-k^2}}{2} \tag{10}$$

The geometry is shown in Figure 2.

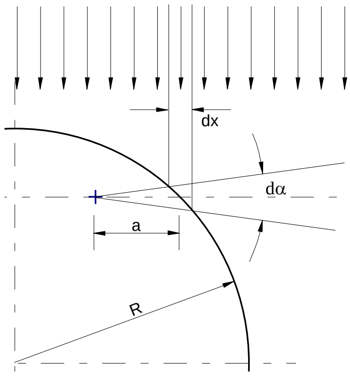

Figure 2 Rays representing a plane wave are reflected by the cylinder.

The intensity of the inbound plane wave may be characterized by an area related sound power level of L''_W – it is the same numerical value as the sound pressure level L_p.caused by this wave. The length related sound power level of the mirror image source L'_W results from the identity of the sound energy radiated from the source and the energy reflected from the surface element into the cone $d\alpha$.

$$\frac{d\alpha}{2\pi} \cdot 10^{0.1 \cdot L'_W} = dx \cdot 10^{0.1 \cdot L''_W} \tag{11}$$

Introducing (9) and rearranging results in the emission of the mirror image source.

$$L'_W = L''_W + 10 \cdot \lg\left(\frac{\pi R \sqrt{1-k^2}}{l_0} \right) \tag{12}$$

This allows to describe the reflection of a plane wave at a small element with bredth dx of the cylindrical shell by a cylindrical wave radiated from a line source parallel to the cylinder axis. This reflected wave element is bounded by the two straight rays from the source to the edges of the element dx.

With known direction of the incoming ray the direction of the reflected ray is given by (6) or (7), the distance of the mirror image source from the reflection point by (10) and the acoustical emission of the mirror image source by (12).

Figures 3a – 3c show the distribution of sound pressure levels around a reflecting cylinder with a diameter of 10 m. The incoming plane wave has been approximated by the sound field produced by a point source 300 m away.

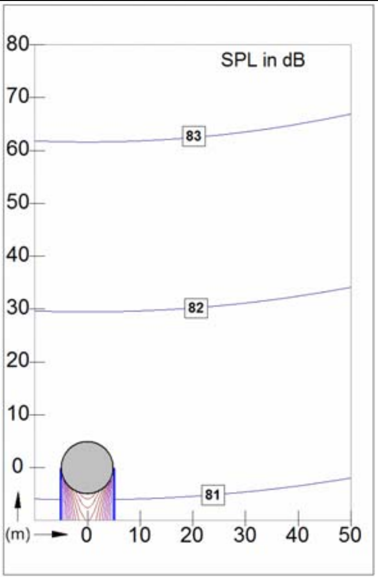

Figure 3a Absorbing cylinder
(only direct sound field)

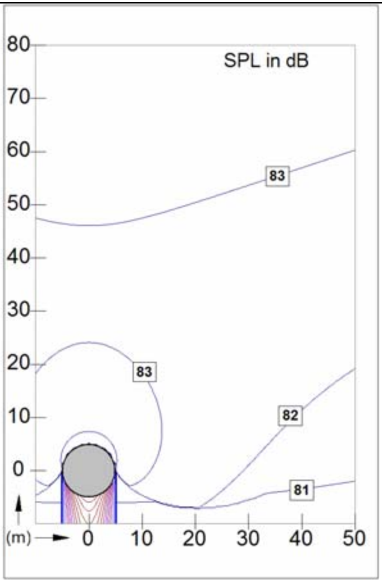

Figure 3b Reflecting cylinder
(direct and reflected sound field)

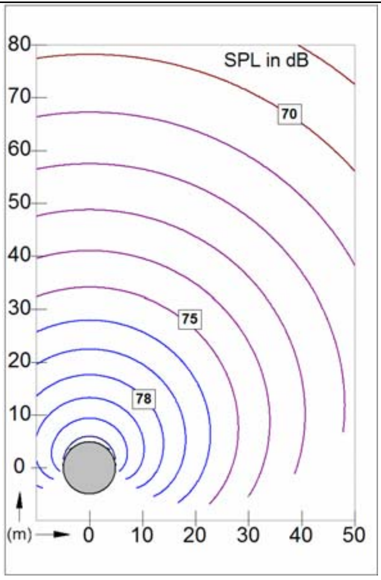

Figure 3c Sound field from
reflection

Figure 3a shows the distribution of sound pressure levels without, Figure 3b with reflected sound. The energetic difference of these two level distributions shown in Figure 3c is therefore caused by the reflected sound.

Sound radiated by a point source and reflected at a cylindrical surface

According to ISO 9613-2 sound sources are represented generally by point sources – extended sources are subdivided in pieces small enough to be replaced by point sources. This method is well-tried and has proven to be the best solution for software implementations, because independent of the complexity of different sound sources the propagation calculations are only linked once correctly with the point source.

Figure 4 is a projection to a plane perpendicular to the cylinder axis. A ray cone with aperture diameter dx at the reflection point and aperture angle $d\beta$ will be widened additionally by the curvature of the reflecting surface. The aperture angle of the reflected ray is $d\alpha + d\beta$. The position of the mirror image source can be derived from geometry, because the aperture diameter of the reflected ray is also dx. With the distance of the source d_s the aperture angle of the inbound ray cone is

$$d\beta = \frac{dx}{d_s} \qquad (13)$$

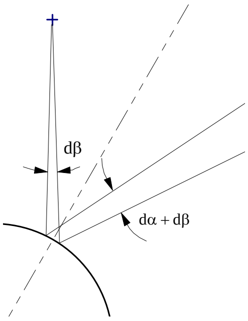

Figure 4 Additional widening of the reflected ray cone

Introducing (9) we get the aperture angle

$$d\alpha + d\beta = dx \cdot \left(\frac{2}{R \cdot \sqrt{1-k^2}} + \frac{1}{d_s} \right) \qquad (14)$$

From this angle follows

$$d\alpha + d\beta = \frac{dx}{b} \qquad (15)$$

and the distance b by introducing (14) in (15)

$$b = \frac{R \cdot d_s \cdot \sqrt{1-k^2}}{R \cdot \sqrt{1-k^2} + 2 \cdot d_s} \qquad (16)$$

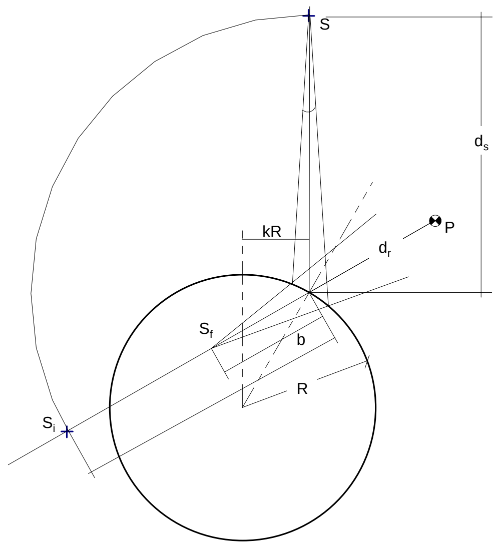

Figure 5 Geometry of a ray cone reflected by a cylindrical surface

If we regard an inbound ray cone with cross section dx * dx, the cross section of the reflected ray is the same direct behind the reflecting surface. The reflected ray cone is characterized by a lateral aperture angle of $d\alpha + d\beta$, while the vertical aperture angle is still $d\beta$.

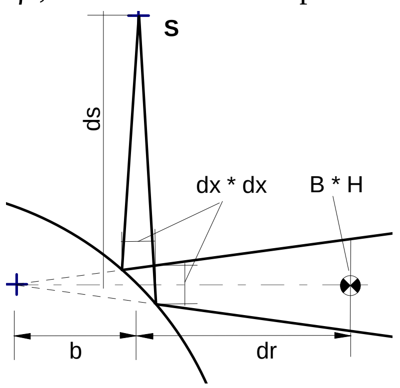

Figure 6 Geometry of rays in a projection parallel to the cylinder axis

The cross section (dx)² of the ray cone at the reflecting surface with distance d_s from the source S is widened to a cross section B * H at the receiver position with distance d_r from the reflecting surface. These geometrical relations can be expressed as

$$B = dx \cdot (1 + \frac{d_r}{b}) \qquad (17)$$

$$H = dx \cdot \left(1 + \frac{d_r}{d_s}\right) \qquad (18)$$

The cross section F_Z of the ray cone reflected at the cylinder is at the receiver position

$$F_Z = (dx)^2 \cdot \left(1 + \frac{d_r}{d_s}\right) \cdot \left(1 + \frac{d_r}{b}\right) \qquad (19)$$

If the cylinder is replaced by the tangent plane the cross section of the reflected ray cone F_E would be

$$F_E = (dx)^2 \cdot \left(1 + \frac{d_r}{d_s}\right)^2 \qquad (20)$$

Therefore the additional attenuation caused by the curvature of the cylinder is

$$A_{curv} = 10 \cdot \lg\left(\frac{1 + \frac{d_r}{b}}{1 + \frac{d_r}{d_s}}\right) dB \qquad (21)$$

Introducing (16) in (21) eliminates the distance of the mirror image source b and we get

$$A_{curv} = 10 \cdot \lg\left(1 + \frac{2 \cdot d_s \cdot d_r}{R \cdot (d_s + d_r) \cdot \sqrt{1 - k^2}}\right) dB \qquad (22)$$

Independent of the real incline of the ray the variables d_s and d_r are the lengths of these rays projected to a plane perpendicular to the cylinder axis.

With a point source far away ($d_s >> R$) expression (22) can be simplified to

$$A_{curv} = 10 \cdot \lg\left(1 + \frac{2 \cdot dr}{R \cdot \sqrt{1 - k^2}}\right) \qquad (23)$$

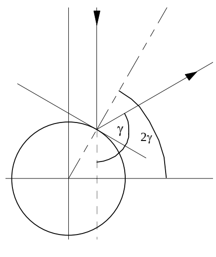

Figure 7 Relation between the gradient of the tangent normal and the scattering angle

From Figure 7 follows that the scattering angle caused by the reflection at the cylinder surface is twice the gradient of the tangent normal with

$$\gamma = 2 \cdot \arctan\left(\sqrt{\frac{1}{k^2} - 1}\right) \qquad (24)$$

Extracting k and introducing it in (23) gives

$$A_{curv} = 10 \cdot \lg\left(1 + 2 \cdot \frac{d_r}{R} \cdot \frac{1}{\sqrt{1 - \frac{1}{1+\tan^2\left(\frac{\gamma}{2}\right)}}}\right) \qquad (25)$$

The diagrams in Figure 8 show that this additional attenuation increases infinitely at the outer edge of the cylinder where the scattering angle becomes small. The attenuation is minimal if the axis of the inbound ray and the axis of the cylinder lie in a common plane and the projected scattering angle is 180°. The attenuation increases with distance d_r because even an inbound ray pencil with parallel rays forms a cone after being reflected and the cross section increases with distance.

Figure 9 shows the curves for the values 0.1 and 1000 of d_r/R if the minimum values related to backscattering or 180° are subtracted.

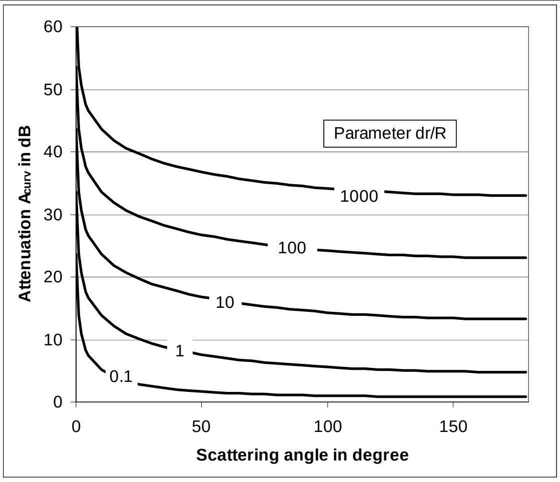

Figure 8 Attenuation in dependence of scattering angle

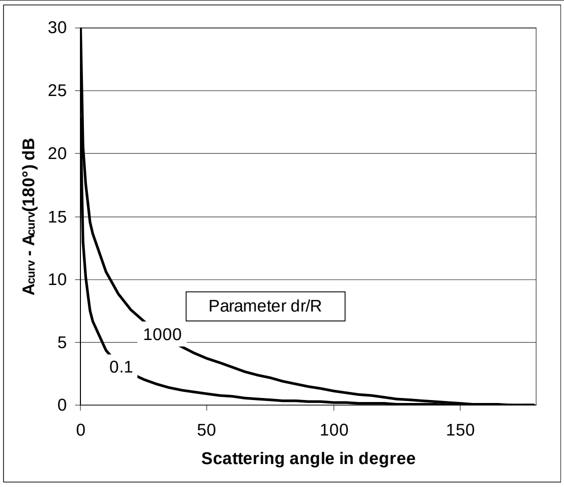

Figure 9 Attenuation normalized with value for backscattering

Integration in the methodology of ISO 9613-2

Given are the source S, the receiver P and the cylinder with radius R, that is acoustically characterized by a reflection coefficient ρ of ist surface. The lower edge of the cylinder is assumed to lie in the ground plane and its axis is vertical. The following steps have to be performed.

1. Determination of the reflection point O and d_s, d_r and k
   The position of the reflection point is determined using the equality of the angle of inbound and reflected ray with the tangent in top view. This top view is also relevant to determine the lengths d_s and d_r for inbound and reflected ray in z-projection. The shortest distance of the straight SO from the cylinder axis divided by the cylinder radius is k.

2. Checking the reflection condition
   It has to be checked if the reflection conditions according to 7.5 in ISO 9613-2 are fulfilled. Reflections are only taken into account, if the reflection coefficient of the surface exceeds 0.2 and if the reflection condition equation (19) of ISO 9613-2 is fulfilled regarding the relevant wavelength. To check this condition the cylinder is replaced by the tangential plane with bredth 2R and same height.
   In future developments it could be advantageous to determine the percentage of the Fresnel Zone that contributes to the reflection and so to smoothen the transition from reflecting to not reflecting conditions.
3. Determination of the additional attenuation A_{curv} from (22).
4. Determination of the partial level caused by the reflection at the receiver as contribution of the mirror image source with a location determined from the tangential plane as mirror plane. The sound power level of the mirror source is determined from
   $$L_{W,im} = L_W + 10\lg(\rho) dB + D_{Ir} \qquad \text{(Equation 20 in ISO 9613-2)}$$
   and the partial level of the reflected sound is
   $$L = L_{W,im} - A_{curv} - A \qquad (23)$$
   with
   $$A = A_{div} + A_{atm} + A_{gr} + A_{bar} + A_{misc} \qquad \text{(4 in ISO 9613-2)}$$
   (Abbreviations and variables see ISO 9613-2). The attenuations A are calculated taking into account the complete 3-dimensional propagation path from source to receiver.

Important hint:
The level caused by sound reflected at a cylindrical shell cannot be determined if the circle of the cylinder cross section is replaced by a polygon and the reflection is checked and calculated for these panels forming the shell separately.

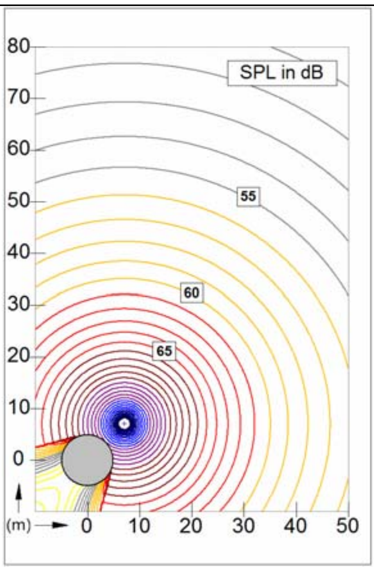

Figure 10a Absorbing cylinder (no reflection)

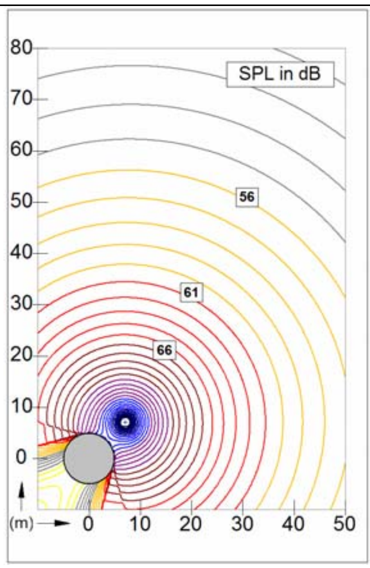

Figure 10b Reflecting cylinder (direct and reflected sound)

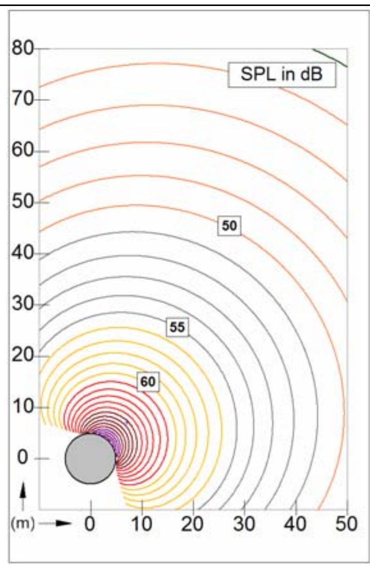

Figure 10c Level distribution from reflected sound only

Figures 10a and 10b show the distribution of levels calculated applying the methodology described above. The radius of the cylinder is 10 m and a point source in 1 m distance from the cylinder shell is radiating with a sound power level of 100 dB. The distribution Figure 10a has been calculated with no reflection and Figure 10 b with reflection at the cylinder. The energetic difference of these levels Figure 10 b and 10 a is presented in Figure 10 c – it is the sound level distribution of the reflection alone.

Test case to check the correct calculation with computer programs – quality assurance

According to DIN 45 687[5] test cases shall be provided to with each calculation standard or guideline that the user of a software can control the correct implementation.
The following simple scenario is such a test case.

Source:
x/y/z → 7.071/17.071/1000
L_W → any, e. g. 100

Cylinder:
Center → 0/0, radius 10, height 2000
Reflection coefficient 1 (absorption coefficient 0)

Vertical tangential plane E:
0/14.17 – 14.17/0
Height 2000
Reflection coefficient 1 (absorption coefficient 0)

Receivers:
y/z → 7.071/1000
x → 7.07 + A with A 1, 2…..10, 12….50

The heights are large to avoid ground influences.
Part 1 – The tangential plane is active and taken into account as reflector– the cylinder is deactivated or deleted.

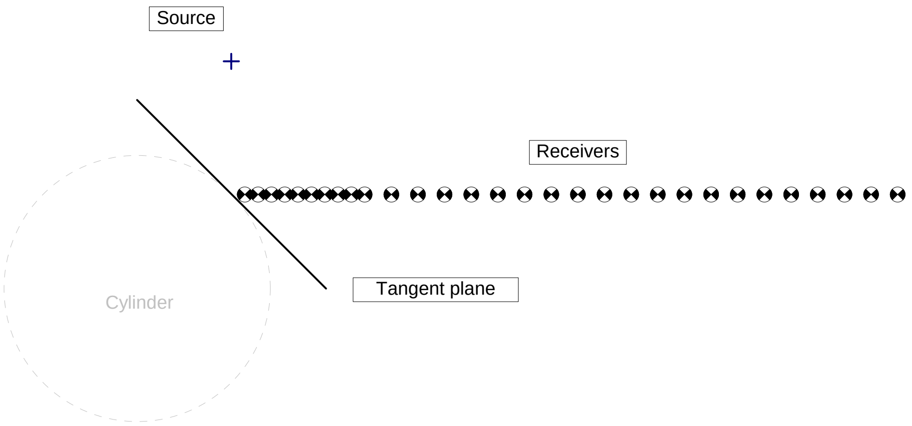

Figure 11 Scenario with only the tangential plane as reflector

Step 1:
Calculation of levels $L_{E,mR}$ with reflection at tangential plane E (without cylinder)
Step 2:
Calculation of levels $L_{E,oR}$ without reflection at tangential plane E (only direct sound)
Step 3:
Calculation of levels $L_{E,R}$ representing the reflected sound at the plane as level difference of $L_{E,mR}$ and $L_{E,oR}$

Part 2 – The cylinder is active and taken into account as reflector – the tangential plane is deactivated or deleted.

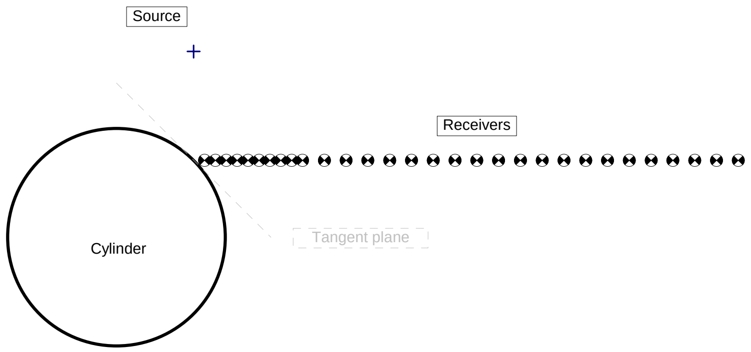

Figure 12 Scenario with only the cylinder as reflector

Step 4:
Calculation of levels $L_{Z,mR}$ with reflection at cylinder Z (without tangential plane)
Step 5:
Calculation of levels $L_{Z,R}$ representing the reflected sound at the cylinder as level difference of $L_{Z,mR}$ and $L_{E,oR}$
Step 6:
The additional attenuation A_{curv} caused by the curvature of the cylindric reflector is the arithmetic difference
$A_{curv} = L_{E,R} - L_{Z,R}$

Each receiver produces a point in the following diagram Figure 13. The poygon line connecting these points should completely remain inside the frame given by the upper and lower bold lines in the diagram.

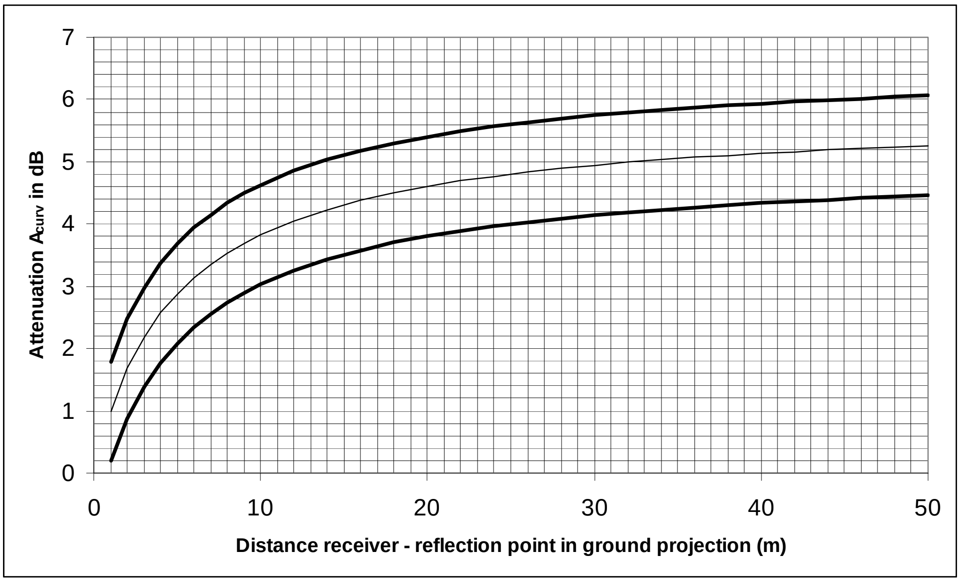

Figure 13 Diagram to check the correct implementation of the described methodology in computer programs.

The interval given by these two curves is necessary to apply the procedure even in cases where the calculated receiver levels are rounded to one decimal (1/10 dB).
Figure 14 shows the result of such an analysis, where the levels rounded to one decimal have been used.

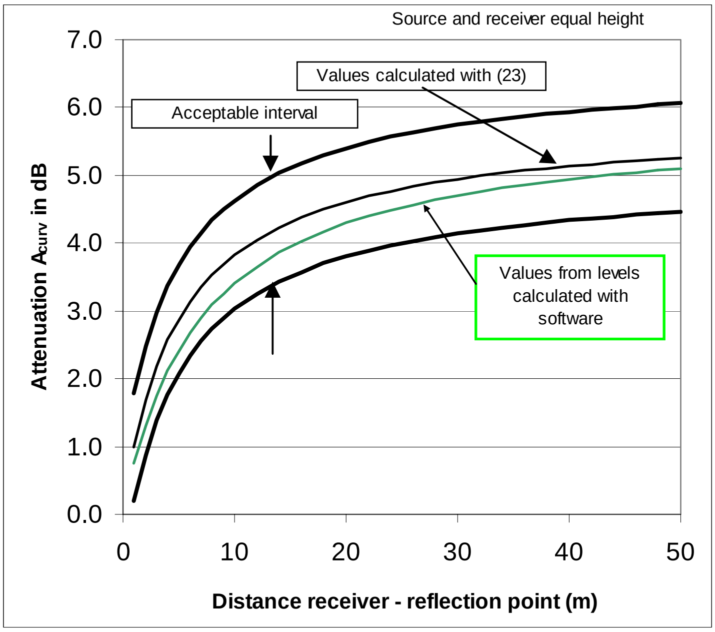

Figure 14 Practical example where the models Figures 11 and 12 have been applied

Practical applications

The application of cylinders is often necessary if noise levels for planned scenarios shall be calculated. Especially in commercial and industrial environments such devices like tanks, vessels, ovens, stacks or even buildings with complete or partial spheric outline can play an important role. Figure 15 shows such a model of an industrial plant.

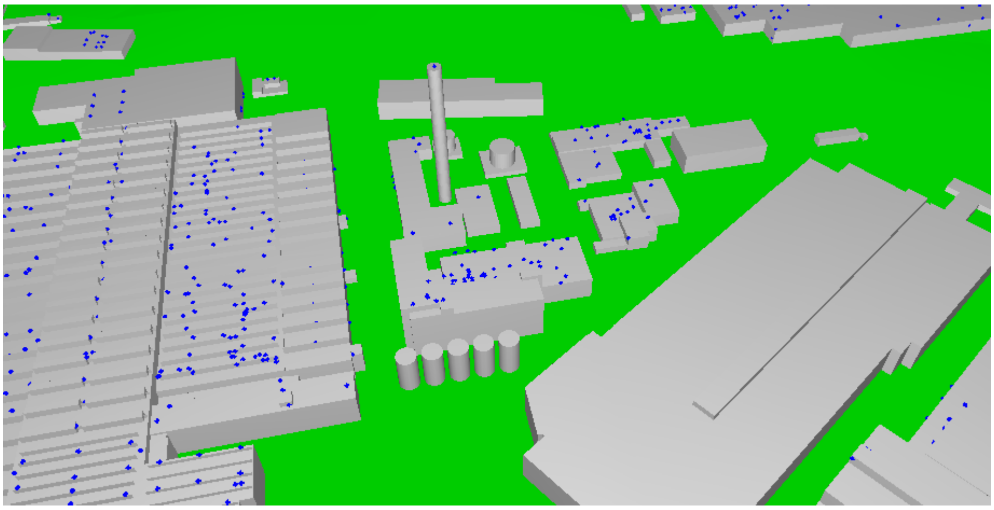
Figure 15 Model of a power plant on an industrial estate (car production)

The methodology presented has been developed and was implemented in a computer program for noise prediction[6] some years ago and experience shows that it can be recommended generally to be used in the calculation of sound propagation with ISO 9613-2

References

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